Change in the Concept of Equilibriums and Reformulation for Newton's Second Law in Presence and in Absence of Gravitational Field

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Abstract-Newton's second law which deals with measurement of force that acted on a body by change in velocity upon time taken or rate of change of the linear momentum with time represented mathematically by F=ma, is a way to measure force acted on that body in absence of external force or external force exerting field. But this formula can't measure force completely in presence and in absence of external force exerting field such as in Gravitational field, although instead of F=ma there it is replaced by F=mg. So, a complete way to measure force acted on a body is of common essence. For the purpose I have intended forward to formulate a complete way to measure force acted on a body in such Gravitational field and in absence of such field. It is denoted by formula $F=mg\sin\theta+mV_0V_f/R$ in presence of Gravitational field and F=ka in absence of such field where, letters have their usual meanings.

Index terms- force measurement, equilibriums, Gravitational field, Newton's laws, inertial frames of reference, terminal velocity, coefficient of viscosity.

Introduction:

Almost all of the experiments on Mechanics till date are based on Newton's laws of motions in terms of force measurement. Particularly Newton's second is standing as the Pillar of Mechanics because it deals from micro to macro level to calculate the net force presence on a particle and on a body with huge mass. Furthermore the concept of Newton's first law is only the part of Newton's second law. Some of the examples implementing these laws are Stoke's law, Millikan oil drop experiment, etc. One thing we can observe in Mechanics is that whenever the Newton's laws of motion deals to illustrate the phenomenon or the nature of balancing force occurs, there will come the concept of Equilibriums to deal the balancing of the forces those get exerted on a body. In above mentioned experiments also the concept of Equilibrium is referred from Newton's First law that states that all inertial frames deal acting no force. It means whenever a body is allowed to move (in absence of external force) in uniform velocity, it will continues its motion permanently i.e. due to inertia of motion (in absence of external force) and on similar way if the same body is allowed to have a rest on a certain point it will too permanently stay at same point forever i.e. due to inertia of rest(in absence of external force). From this kind of view the concept of Equilibrium is divided into two parts. The phase moving with uniform (following inertia of motion) velocity is referred as Dynamic equilibrium and the phase having no motion (following inertia of rest) is referred as Static equilibrium. But the matter is that though these phases achieved by the same body are different but both of these posses zero resulting forces at all. In the experiments mentioned above, a term 'terminal velocity' which allows to fall the oil drops or piece of iron ball in downward direction uniformly from a particular level of medium, is referred as the phase of dynamic equilibrium where it is said that the net down ward force(weight W) is equalized by the up-thrust (U) and the viscous force(F) that acts on surface of falling body (as like a resistive force) inside the medium. i.e. W=U+F.

Body:

Now the matter arise here that, how this phase of gaining uniform velocity can be referred as the phase of Equilibrium or say the phase of balancing forces? According to Newton's first law there must not be any other external force that could come into action to have the state of Equilibrium but here the viscous force which resists the motion of the body is representing as an external force which is taken wide openly. Is it appropriate to refer such phase as a balancing state of forces? And more strongly if we put the body directly at a point where it gained terminal velocity previously, will the body remains stationary at same point where we put it? No, it will not remain at the point we placed it on certain level of medium. It will slowly comes down at the bottom. It means that there must be some quantity of force that drags the body downwards due to Gravitational force exertion to that body by earth. But from the Newton's first law that body must be remained at the same point where we put it obeying inertia of rest. The answer to these questions can't be given by the Newton's laws of motion. What mattered here is that Newton's laws of motion can't be applied in presence of external force and in presence of such external force exerting fields such as in Gravitational field with respect to mass and in Electrostatic field with respect to charge to deal the phase of Equilibrium of forces. If we perform the same experiments in absence of Gravitational field (say space for consideration) then the body or oil drop or the spherical iron ball will not gain any motion though we put it on the surface of medium or liquid there the body does not follow as it follows in Gravitational

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field. It means the body will not fall from the medium though if we let it on surface of medium.

So, there must be change in the concept of Equilibriums to deal with the balancing state of forces in presence of external forces and external force exerting fields. The two types of equilibriums say Static and Dynamic have respective regions for application. As Static equilibrium deals with balancing of the forces those just get cancelled out with zero displacement of body, it must be applied in Multi-body system at a phase when the body remains at rest though it has interactions of different forces but the Dynamic equilibrium that deals with balancing of forces with uniform displacement of body it can be possible only when the body move in orbit with orbital velocity though there is change in direction continuously. But such changing of direction will not be there if we trace different perpendicular lines from the different points on the circumference of earth surface. It seems quite odd but in reality moving on orbit is actually due to balancing the value of centripetal and centrifugal forces in against of one another with uniform angular displacement of the body is due to the dynamic equilibrium. But no such concepts of Equilibriums can be applied on Single body system because, a single body neither get interacted with other forces nor will it have effect of its own. Here, let us introduce the terms Single and Multi-body systems.

(1)Single body system: It is the system where only one body exists and the body must not be in influence of other disturbances like Gravitational field, Electrostatic field, etc. Perhaps it may not be possible or say such body system region could not be possible in reality if the entire universe is holding due to Gravitational forces and their interactions with different masses. It may obey Newton's first law of motion, but not the second law. One will surely get puzzled by this statement but no matter I will show here how this could be possible.

Let us take a mass 'm' having its magnitude as 1 kg on near about the surface of earth. Its weight will be 10N on the surface of earth, if we take it away from the surface of earth to the infinity the magnitude of its weight will slowly goes on decreasing and finally at infinity its weight will be zero following Newton's law of Gravitation .i.e. $F = GMm/r\hat{}2$. Here infinity represents a region what we call as space now where gravity is assumed to be zero. It shows that 1kg mass will have weight 10N approximately on the surface of earth and 0N in the infinity. It means a man carrying that mass 'm' on earth will continuously get suffered by 10N tension and the same man carrying that same mass will get no tension in the infinity. Now if we go on increasing the mass from 1kg to 1ookg, the man can't carry the masses as the weights of those masses goes on increasing on the surface of earth but there will be no effect on changing the magnitude of masses in infinity, the man will feel as if there is no change on masses. He can take all these masses anywhere as his wish. Say that man allows mass 'm' to move with some constant mechanical force (F) acting on that mass 'm'. The mass will move on constant accelerating (a) condition. If the mass 'm' is replaced by another mass of greater magnitude then at this condition, also the mass must move on same acceleration(a) showing that mass is independent to the force applied on a body in space but depends upon the change in velocity with time(a). So, force applied will be directly proportional to the acceleration only. Thus,

$F \alpha a$

$.: F = ka$ on integrating this equation with respect to velocity (not with distance because, distance covered can't be isolated from time that it required), we get

$$\int F.dv = \int k/t(V_1 - V_2) \qquad (1)$$
$$.: Power(P) = k/2t(V\hat{}2)$$

Where, k is the value that will be universal constant, if we take unit of force as Newton (N)= $kgm/\sec\hat{}2$. This k will be independent on the magnitude of mass bearing the force in absence of Gravitational pulling force. And V_1 and V_2 are the final and initial velocities of the mass 'm' within time interval't'.

(2)Multi-body system: It is the system where two or more than two bodies co-exist and these bodies make their influences of interactions effective with one another. Here influences of interactions means the effects of Gravitational fields, Electrostatic fields, etc. It is the real system where we are existing. Our universe, galaxies, solar system, etc. are good examples. It's the system where Dynamic equilibrium and Static equilibrium holds at respective phase of motions. From the Newton's law of Gravitation all the masses in the entire Universe are guided by the Gravitational forces those setup immediately in presence of Gravitational fields. And all the lower masses are revolving around the bigger masses such as satellites are revolving around the planets, planets are revolving around the sun and sun is following the same process around the Galactic centre. If each and every massive bodies are rotating with respect to their mass and Gravitational pulling capacity is true, it can't be easy to find the region where such pulling or Gravitational forces are ineffective. It means every region of space is under the influence of Gravitational fields of certain massive bodies indicating that there is no region in space where the inertial frames of references do really exist. Let's have another such kind of illustration that default the application of Newton's laws of motions focusing basically on his second law that states that time rate(dt) of change of linear momentum (dp)is directly proportional to force applied (F)and takes place in the direction of force applied, represented mathematically by F=dp/dt=mdv/dt=ma. Thus, F=ma is the mathematical equation that is used in order to measure the quantity of force that a body possesses or a body is applied with, taking no reference to Gravity. And F=mg is also a similar equation used in order to measure the quantity of force taking reference to Gravity. But this equation is not also a complete format equation that can measure the net force acted on the body.

Let a body with mass 'm' is thrown with certain velocity 'v' in absence of Gravitational field, it follows the inertial frame of reference, tracing the linear path. If the same mass 'm' is projected in horizontal form in presence of Gravitational field say taking earth as source for Gravitational field with same velocity 'v' then it will not follows the inertial frames of reference, tracing the curve path depending upon the limit of velocity we have thrown. If the limit of velocity is well enough to meet at the level of orbital velocity then the mass 'm' will continuously move on uniform velocity tracing the circular path that introduces the maintenance of centripetal and centrifugal forces. If the velocity is greater than the orbital velocity the path traced will be parabolic around the earth and if the velocity is lesser than orbital velocity the path traced by the mass 'm' will also be parabolic towards earth. One thing we need to take care is that below the orbital velocity, a reducible quantity due gravity(g) reduces the direction of the body towards the earth's centre so the path traced will be parabolic. Similarly, above the orbital velocity body tends to escape due to increasable quantity so the path traced will be either parabolic or hyperbolic. Here the terms reducible and increasable quantities are respective components of gravity(g) that will be notify later on. It is the case whenever the body is thrown at horizontal form. If the body is projected in vertical form, it will reach at certain height and come to same point again. But if we project or let to move the body in any direction then there the body will itself gain acceleration or we need to apply force to gain acceleration to that body. With reference to this kind of phenomenon I have formulated a formula that could correctly measure the net force presence on the body in a sense of force applied on that body or force gained by that body. We must consider that the horizontal line passing along the surface of earth as the reference line that separates the condition of force applied or force gained.

The parts above this horizontal line represent the sense of force applied and the parts below this line represents the sense of force gained. But this line itself represents the sense of force applied. This horizontal line is nothing other than the line that passes tangentially through the earth surface on each and every points of its circumference. Now let's calculate the force in any direction:

In terms of force applied:

As the horizontal line itself represent the sense of force applied, at which all the masses that come to orbit the earth must lie or must trace the path along this line, they must have an orbital velocity to remain along this line such that the angle between them remains zero degree. It refers the body to be in equilibrium maintaining the centripetal and centrifugal forces in balance to one on another. It is the condition of Dynamic equilibrium where these centripetal and centrifugal forces are balanced to each other with uniform angular displacement of mass 'm'. At this condition weight of mass 'm'(mg) is balanced by force (mv_0^2/R) i.e.

$$mg = mv_{\circ}\text{^}2/R$$

$$\therefore a_1 = g = v_0\text{^}2/R \quad (2)$$

Where , $v_{\circ}$ is the orbital velocity and R is the radius of earth. Now let us suppose that the mass 'm' be projected or moving above this horizontal line making certain angle to this line say angle θ. At this time mass 'm' will now either form projectile motion or it will escape from the surface of earth depending upon the limit of velocity. It is a complex state which is not defined by present science in terms of force measurement completely though it has been illustrated under the equation of motion in projectile chapters. Here as soon as the mass 'm' is projected or allow to move, there comes the role of Gravitational force in terms of gravity(g) to affect the motion of that projectile. At this condition factor affecting the acceleration due to gravity(g) is only the distance travelled(h) from a fixed point that correspondingly even affect the height travelled(p) as shown in figure below:

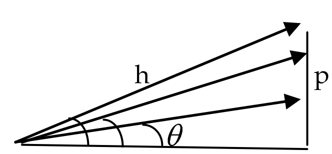


Figure 1: showing the effect of gravity to mass 'm' after projection.

As the body tends to gain height(p) with increasing order of θ then acceleration due to gravity stands in opposition i.e. θ increases causing decrease in the distance travelled(h) by mass 'm' for a constant force. But a fixed gravitational mass(here earth) has fixed value of gravity(g) within a certain region of its gravitational field. So in this case 'g' will be resolved accordingly with $\sin\theta$. Therefore, acceleration that needs to share with 'g' by mass 'm' can be taken as:

$$a_2 = g\sin\theta \quad (3)$$

Here, for the range of $\sin\theta$ at (0-180)˚ its value will be positive and positively maximum at sin90˚ so we need to apply more force in terms of gaining acceleration at this case. But up to present this g is taken 'g' only without componential division i.e. without resolving with sine factor. Thus, if v be the instant velocity of body, net acceleration acting on mass 'm' in the force applied direction will be

$$a = a_1 + a_2 = v^2/R + g\sin\theta \quad (4)$$

In terms of force gained:

Whenever the mass 'm' moves in downward direction we need not need to apply force but gains acceleration itself where range of θ limits within(180-360)˚ whose value will be negative and negatively maximum at θ=270˚. At this condition the body will be under free fall equalizing the value of 'g' with 'v^2/R'. In this condition the numerical value doesn't get equalize to each other if mass 'm' is allowed to fall from certain height, le β be any numerical value kept on RHS that equalizes RHS and LHS. So the exact equation will be

$$g = \beta v^2/R \quad (5)$$

This particular case will be obtained only in absence of air resistance. In presence of air or any medium that obstacles the motion of the body, g>βv^2/R. And the difference between g and βv^2/R measures the net anti force caused due to the medium. It is called as up-thrust. This equation (5) is exactly the same form as it is in above equation (4). Only net change in acceleration 'a'=0 in absence of air resistance. But the form is same if there is any disturbance. If the value of g<βv^2/R, we need to analyze out that an extra source is applying force to the body in same direction of motion. Such conditions will be deal in more detail at below for calculating the value of β.

From these above two cases we can generally represent out a figure that can illustrate the meaning more clearly.

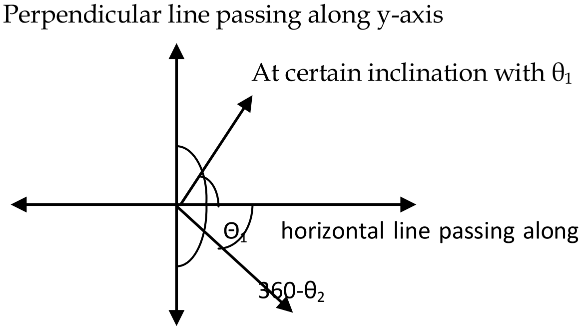


Figure 2: showing the componential division of 'g' with different angles at different positions

On similar way this kind of phenomenon can more clearly be shown by the different motions of mass 'm' in different way with or without earth as a source for Gravitational pull as like in figure below:

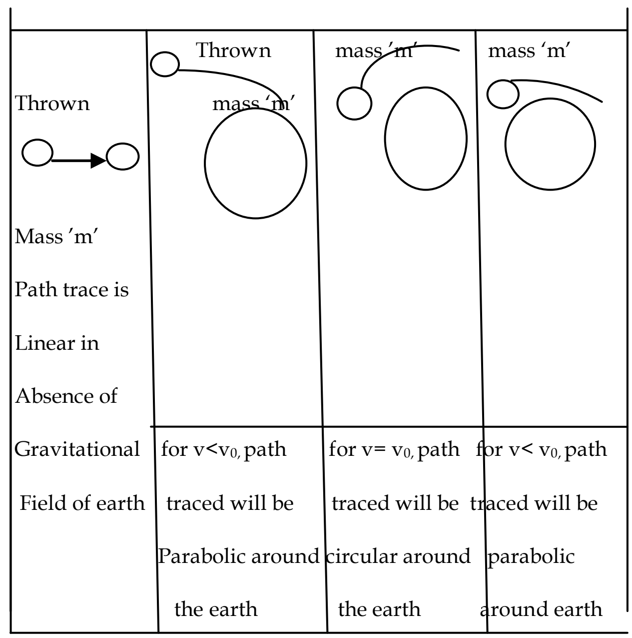
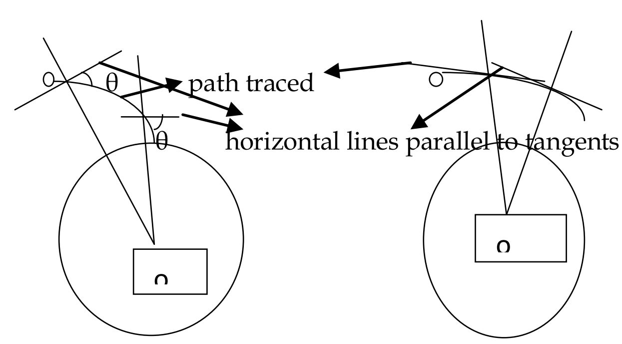


Figure 3: Showing the path followed by mass 'm' in presence or absence of Gravitational field thrown with different velocities

In this above figure, the paths traced by mass 'm' is different with vary in limit of velocities, these paths continuously deviate their directions in presence of Gravitational field. Here, we can use the equation (6) to measure the net acceleration acting on mass 'm' more correctly. For this purpose we need to calculate the net change in direction i.e. θ in every points of the path it follows. What geometrically shows that the circular and parabolic paths (near orbital velocity and below it) will have no change in θ with respect to each horizontal lines those passes tangentially at every points on the surface of earth but the parabolic path around the earth will have continuous change in θ . For our experimental purposes we need to calculate θ to deal for such condition. Let's deal with parabolic and circular paths with their illustrations below in figure:



passing from the surface of earth

Figure (a) figure(b)

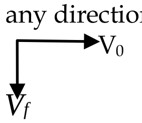
Figure 4: Showing that there will be no change in θ for both

Parabolic (fig.a) and circular(fig.b) paths traced by mass 'm' on moving.

These are the cases when the mass 'm' is projected with different velocities. This shows that below the orbital velocity and below or near about θ=0˚, if any mass is projected then there will be always the conservation of kinetic acceleration (βv ^2/R) and potential acceleration ($g\sin\theta$). But when there is presence of external source that provides the force to the mass 'm' then such conservation will be break and force calculation can be done by using the equation(4) for any kind of motion on earth surface or say on any Gravitational surfaces.

Calculation of value of β:

Suppose, mass 'm' is allowed to fall from near to the infinity towards the centre of earth. On falling downwards, velocity goes on increasing order as the distance gets shorter and shorter from the centre of earth. On falling downward, what we need to note is that as the body falls down at any particular point of falling, the velocity measurement gives the velocity for orbital velocity i.e. the tangential velocity required to orbit the earth surface will equalize this particular point velocity. It will be more clear from the figure below where, v_0 is the orbital velocity required at a particular point to gain kinetic acceleration v_0 ^2/R and V_f is the falling velocity acquired by mass 'm' at the same point. And this V_f is taken as the instant velocity in any direction for the final procedures.



Showing that $V_0=V_f$

But if the mass 'm' is allowed to fall from certain height, we can't have $V_0=V_f$ at any particular point of falling. It means for the body falling from the infinity that will possess the velocity limit in the form of orbital velocity else will posses less than that at any particular point of falling. This shows that for the body falling from infinity will have value of β=1 and the one falling from any height will have β other than 1. One thing we need to consider out is that *a body which may be either in the rest or in the motion at any particular region of gravity, will have the tendency to be get equalized to the value of gravitational mass at that region*. Thus, β must be greater than 1 for falling from general height. And same value of β will be there that neglects the direction of motion at particular level of gravity because, θ recovers the direction of velocity. But in terms of magnitude β depends upon the velocity of the body. It means β depends upon the value of instant velocity in comparative to the value of orbital velocity. So, value of β suits to the ratio of Vo to the V_f i.e. $\beta=V_O/V_f$ So, the actual force measuring formula will be

$$F=(V_0/V_f)mV_o{}^{\wedge}2/R+mg\sin\theta$$

$$or, F=m(V_0V_f)/R+mg\sin\theta \quad (6)$$

Again on integrating equation (6) with respect to velocity, we will get

$$\int F.dv=\int m\{(V_0V_f)/R+g\sin\theta\}.dv$$
$$.:P=mV_f\{V_0V_f/2R+g\sin\theta\}. \quad (7)$$

Conclusion 1:

Points to be noted from the equations (1), (6) and (7) are majorly listed below:

a) If the body gains no velocity at a particular region of gravity but remains stationary at that case V_0Vf/R will be 0, but a=g indicating that force at that time gives weight of the body. It is the real condition of static equilibrium.
b) A body either in rest or a moving in any direction will always have a tendency to remain with the force that gets exerted by Gravitational pull or simply weight of that body.
c) There is no need to be change in momentum to have a force magnitude on a body. So, a constant momentum will have constant force. It doesn't link up with time factor. It means a body resting at any point will remain at that point forever with a magnitude of force as its weight up to infinite time and similarly the body orbiting the Gravitational mass will remain at position forever up to infinite time.
d) Power is the ultimate step that we can obtain or calculate, not the energy which avoids time link to cover a distance. And mass is the factor that linkup with force only in presence of Gravitational pull but is independent in absence of such pulling.

Thus, in general sense, the net force gained by body or applied on a body is always comparative to the value of gravity 'g' of any Gravitational masses on its surface. And the force gained by the body in any direction is calculated as $F=m(V_0V_f)/R+mg\sin\theta$ where terms have their own usual meanings. Also, there is no need of motion to have a force magnitude on a body.

One more illustration that needs to be change from present concept which is dealing with classical concept of Newton's laws of motion:

Millikan's oil drop experiment:

In Millikan's oil drop experiment in first case there the oil drops are allowed to fall with terminal velocity and this phase is referred as the phase of Dynamic Equilibrium saying that the weight (w) of each falling oil drops is balanced by the air particles with up-thrust (u) as there is no change in velocity to have a force magnitude on oil drops from Newton's second law. And on falling each oil drops suffer a constant resistive force that opposes the motion of oil drop naming this force as Viscous force(F_1). Thus the weight(w) of each oil drop is balanced by sum of these up-thrust(u) and viscous force(F_1). i.e. Weight(w)=viscous force(F_1)+up-thrust(u)

If σ be the density of air, V_1 be the velocity acquired by the oil drop, r be the radius of oil drop, η be the coefficient of viscosity of the medium and ρ be the density of oil drop then the actual equation to balance the weight of the oil drop is given by $Mg=6\pi\eta rV_1+4/3\pi r^3\sigma g$ (8)

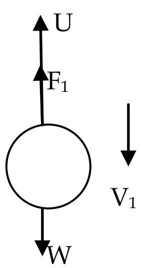


Figure 5: showing that oil drop is falling under terminal velocity V_1

But from the equation (6) above net force that opposes the motion of oil drop will be

$$F = m(V_0V_f)/R + mg\sin\theta$$

$$or, F = m(V_0V_f)/R - mg \qquad (9)$$

here $\sin\theta$=-1, m is the mass of the oil drop an $V_f = V_1$. In equation (8) the meaning of up-thrust (u) and in (6) the meaning of anti-force(F) is same. So we can arrive at a relation as viscous force and the kinetic force is same . so,

$$6\pi\eta rV_1 = mV_0V_f/R$$
$$or, \eta = 2/9(V_0/R)r^2\rho) \quad (10)$$

Similarly, in second case when such oil drops are allowed to move in upward direction with applying the force (Fe) by electric field in V_2 terminal velocity acting F_2 as viscous force. Then the viscous force will change the direction such as shown in figure below:

$$Fe + U = F_2 + W$$
$$or, Fe = 6\pi\eta rV_2 + mg - 4/3\pi r^3\sigma g$$
$$or, QE = 6\pi\eta rV_2 + 4/3\pi r^3(\rho - \sigma)g \qquad (11)$$

where, Q is the amount of charge that oil drop carry and E is the electric field intensity applied to the oil drop.

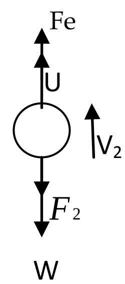


Figure 6: showing that oil drop is moving upward with terminal velocity V_2

Similarly from the equation (6) above net force acting on the oil drop will be $F = m(V_0V_f)/R + mg\sin\theta$ but here Fe is the force applied on oil drop that will move the oil just opposite than that of previous, Fe and U in combination with each other tend oil drop to move in upward direction. So F will be shared by both Fe and U. Therefore,

$$F = m(V_0V_f)/R + mg\sin\theta$$
$$or, Fe + U = mg + mV_0V_2/R$$
$$or, QE = 4/3\pi r^3(\rho g + \rho V_0V_2/R - \sigma g)$$
(12)

On equating the values of QE from the equations (11) and (12) gives the relation

$$\eta = 2/9(V_0/R)r_2\rho \quad (13)$$

Conclusion 2:

Thus, in equations (10) and (13) all values remains the same for same volume of oil drops. So, $\eta\alpha\rho$ only, predicting out that η is the property of the oil drop(falling body) not of the air(medium that allows the body to fall). This shows that viscous force can't be defined as motion opposing force. It links to the medium just for terminal velocity not for value of η, η totally depends upon the nature of body which is allowed to fall. This is the reason for the question, why all bodies of varying densities don't fall with same terminal velocity in the same medium. And thus the value will be different for different bodies with vary in their densities in the same medium though they have equal volumes.

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